Learning-Based Image Super-Resolution

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Outline

1. What is Super-Resolution (SR)?
2. Representative Learning-Based SR Algorithms
3. Limits of Learning-Based SR Algorithms
What is Super-Resolution (SR)?

Sr is a technique that increases image/video details.
Sr vs. Interpolation and Enhancement.
What is Super-Resolution (SR)?
Representative Learning-Based SR Algorithms
Limits of Learning-Based SR Algorithms

Classification of SR Algorithms (1)

- **Interpolation-based**: register + interpolate + deblur

- **Frequency-based**: dealias

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Learning-Based SR
Classification of SR Algorithms (2)

- **Reconstruction-based:** register + weak prior + solve a linear system

  \[ L = PH + E \]

- **Learning-based:** knowledge + inference

(a) Input 24x32  (b) Cubic B-Spline  (c) Baker et al.  (d) Liu et al.  (e) Our method  (f) Original 96x128
Advantages & Disadvantages of Learning-Based SR

+ Require fewer low-res images, even single image!
+ Achieve higher magnification factors (MF)
+ Faster
+ More versatile, e.g., style transfer
  – Work with fixed MFs
  – Performance unpredictable

Input  Train: generic  Train: noise  Train: rects
What is Super-Resolution (SR)?
Representative Learning-Based SR Algorithms
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Classification of Learning-Based SR Algorithms (1)

Based on applications:

- For general images
- For specific images
  - only face/text images
What is Super-Resolution (SR)?
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Classification of Learning-Based SR Algorithms (2)

Based on implementations:

- Indirect maximum a posteriori (MAP)
  \[
  H = \arg \max_{H} P \left( \left\{ \hat{L}_i \right\}_{i=1}^{N} \mid \hat{H} \right) P \left( \hat{H} \right)
  \]
  - Local: Infer the HR image patch by patch
  - Global: Infer the coefficients of the bases for the HR image

- Direct MAP
  \[
  H = \arg \max_{H} P \left( \hat{H} \middle| \left\{ \hat{L}_i \right\}_{i=1}^{N} \right)
  \]
  - Local only
Representative Learning-Based SR Algorithms (1)

Local indirect maximum a posteriori (MAP):

\[ H = \tilde{L} + \hat{H} \]

\[ \hat{H} = \underset{\hat{H}}{\text{arg max}} \ P \left( \tilde{L} | \hat{H} \right) P ( \hat{H} ) \]

\[ P \left( \tilde{L} | \hat{H} \right) = \prod_{k} P \left( \tilde{L}_{k} | \hat{H}_{k} \right), \quad P ( \hat{H} ) = \prod_{\hat{H}_{j} \in \mathcal{N}(\hat{H}_{i})} P ( \hat{H}_{i} | \hat{H}_{j} ) \]

Low Res. Image

High Res. Image
Exemplar Result

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Learning-Based SR
Global indirect MAP: face hallucination only

$$h = \arg \max_h P \left( \{l_i\}_{i=1}^N \mid h \right) P (h)$$

$$P \left( \{l_i\}_{i=1}^N \mid h \right) \sim \exp \left( - \sum_{i=1}^N \xi_i^t Q^{-1} \xi_i \right), \quad \xi_i = l_i - F_i^t P^{(i)} F_i h - \eta$$

$$P (h) \sim \exp \left( - (h - \mu_h)^t \Lambda^{-1} (h - \mu_h) \right)$$
Exemplar Result

What is Super-Resolution (SR)?
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Representative Learning-Based SR Algorithms (3)

Local direct MAP:

\[
H = \bar{L} + H^p
\]

\[
H^p = \arg \max_{H^p} P(H^p | \bar{L})
\]

\[
P(H^p | \bar{L}) \approx \prod_k P(C_k | \bar{L}), \quad P(C_k | \bar{L}) \sim \prod_{i} \psi(B^l_i, B^l_{i+1}) \prod_{i} \Phi(B^l_i, B^h_i)
\]
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Exemplar Result
Using manifold learning techniques:

1. For each LR patch

2. Enforcing local compatibility and smoothness constraints between adjacent HR patches.
Exemplar Result

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Learning-Based SR
### Do limits exist for Learning-Based SR?

<table>
<thead>
<tr>
<th>Input \ Correct</th>
<th>12 × 16</th>
<th>24 × 32</th>
<th>48 × 64</th>
<th>96 × 128</th>
</tr>
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<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
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<td><img src="image12.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Related Work

Limits of Reconstruction-Based SR Algorithms [2]
What are limits of SR?

- Good SR result: close to the ground truth
- Average performance
Abstract Model of Learning-Based SR

- An SR Algorithm: a mapping \( s \) from LR image (low-dim space) to HR image (high-dim space)
- Average performance: expected risk

\[
R(s) = \int r(h, s(d(h)))p(h)dh
\]

- \( r \): risk function, \( d \): downsampling operator, \( p(h) \): distribution
Problem Formulation

\[ R(s) = g_s(N, m) = \left( \frac{1}{mN} \tilde{g}_s(N, m) \right)^{1/2} \]

\[ \tilde{g}_s(N, m) = \int_h \| h - s(Dh) \|^2 p_h(h) dh \]

\( N \): image size, \( m \): magnification factor, \( D \): downsampling matrix

- Does not help if compute \( g_s(N, m) \) for a particular \( s \).
- Find lower bound \( b(N, m) \) for \( g_s(N, m) \) that is valid for all \( s \).
- Lower bound is indefinite if no assumption on \( p_h(h) \) is made.
Statistics of General Natural Images

- The distribution of HR images (HRI) is not concentrated around several HRIs, and the distribution of LR images (LRI) is not concentrated around several LRIs either.
- Smoother LRIs have a higher probability than nonsmooth ones.

Statistics of specific class of images is unclear.
Theorem 1: Lower Bound of the Expected Risk

\[
\tilde{g}_s(N, m) = \int_h \|h - s(Dh)\|_2^2 p_h(h) dh
\]

**Theorem 1:** \(\tilde{g}_s(N, m)\) is lower bounded by \(\tilde{b}(N, m)\), where

\[
\tilde{b}(N, m) = \frac{1}{4} \text{tr} \left[ (I - UD)\Sigma(I - UD)^t \right] + \frac{1}{4} \| (I - UD)\bar{h} \|^2
\]

\(U\): upsampling matrix, \(DU = I\)
\(\Sigma\): covariance matrix, \(\bar{h}\): mean
Sketch of Proof (1)

\[ \tilde{g}_s(N, m) = \int_h \| h - s(Dh) \|^2 \rho_h(h) dh \]

Choose \( Q \) and \( V \) such that \( \begin{pmatrix} D \\ Q \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = I \). Denote \( M = (U \quad V) \). Perform transform \( h = M \begin{pmatrix} x \\ y \end{pmatrix} \), then
Sketch of Proof (2)

\[ \tilde{g}(N, m) = \int \left\| (U \ V) \left( \begin{pmatrix} x \\ y \end{pmatrix} \right) - s(x) \right\|^2 p_{x,y} \left( \begin{pmatrix} x \\ y \end{pmatrix} \right) \, dx \, dy \]

\[ = \int p_x(x) V(x) \, dx, \]

\[ p_{x,y} \left( \begin{pmatrix} x \\ y \end{pmatrix} \right) = |M| \rho_h \left( M \left( \begin{pmatrix} x \\ y \end{pmatrix} \right) \right) \]

\[ V(x) = \int \left\| V y - \phi(x) \right\|^2 \tilde{p}_y (y | x) \, dy \]

\[ \phi(x) = s(x) - U x \]
Sketch of Proof (3)

\[
V(x) = \int \|Vy - \phi(x)\|^2 \tilde{p}_y(y|x) \, dy
\]

\[
\phi_{opt}(x; \tilde{p}_y) = V \int y \tilde{p}_y(y|x) \, dy
\]

\[
V(x) = \int \|Vy\|^2 \tilde{p}_y(y|x) \, dy - \|\phi_{opt}(x; \tilde{p}_y)\|^2
\]

\[
\|\phi_{opt}(x; \tilde{p}_y)\|^2 \leq \frac{3}{4} \int_y \|Vy\|^2 p_{x,y} \left( \left( \begin{array}{c} x \\ y \end{array} \right) \right) \, dy
\]

\[
\frac{1}{p_x(x)}
\]
\[ \tilde{g}(N, m) = \int p_x(x) \left( \int \|Vy\|^2 \tilde{\rho}_y(y|x) \, dy - \left\| \phi_{opt}(x; \tilde{\rho}_y) \right\|^2 \right) \, dx \]

\[ \geq \frac{1}{4} \int p_x(x) \int \|Vy\|^2 \tilde{\rho}_y(y|x) \, dy \, dx \]

\[ = \frac{1}{4} \int \|Vy\|^2 p_{x,y} \left( \begin{pmatrix} x \\ y \end{pmatrix} \right) \, dx \, dy \]

\[ = \frac{1}{4} \int \|VQh\|^2 \rho_h(h) \, dh \]

\[ = \frac{1}{4} \text{tr} \left( (I - UD) \Sigma (I - UD)^t \right) + \frac{1}{4} \left\| (I - UD) \tilde{h} \right\|^2 \]
Theorem 2: If we sample $M(p, \varepsilon) = \frac{(C_1 + 2C_2)^2}{16p\varepsilon^2}$ HRIs independently, then with probability of at least $1 - p$, 
$|\hat{b}(N, m) - \tilde{b}(N, m)| < \varepsilon$.

$\hat{b}(N, m)$ is the value of $\tilde{b}(N, m)$ estimated from real samples, 
$C_1 = \sqrt{E \left( \left\| (I - UD)(h - \bar{h}) \right\|^4 \right)} - \text{tr}^2 [(I - UD)\Sigma(I - UD)^t]$, 
and $C_2 = \sqrt{\bar{b}^t\Sigma\bar{b}}$, $\bar{b} = (I - UD)^t(I - UD)\bar{h}$. 

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Sketch of Proof (1)

\[ |\hat{b}(N, m) - \tilde{b}(N, m)| \leq \frac{1}{4} |\xi - \mathbb{E}\xi| + \frac{1}{2} |\eta - \mathbb{E}\eta| \]

\[ \xi = \text{tr}(B\hat{\Sigma}_M), \quad \eta = \bar{b}^t\hat{h}_M \]

\[ \hat{\Sigma}_M = \frac{1}{M} \sum_{k=1}^{M} (\hat{h}_k - \bar{h})(\hat{h}_k - \bar{h})^t, \quad \hat{h}_M = \frac{1}{M} \sum_{k=1}^{M} \hat{h}_k \]

\[ B = (I - UD)^t(I - UD), \quad \bar{b} = B\bar{h} \]
Sketch of Proof (2)

\[
|\hat{b}(N, m) - \tilde{b}(N, m)| \leq \frac{1}{4} |\xi - \mathbb{E}\xi| + \frac{1}{2} |\eta - \mathbb{E}\eta|
\]

\[
P(|\xi - \mathbb{E}\xi| \geq \delta) \leq \frac{\text{var}\xi}{\delta^2} = \frac{C_1^2}{M\delta^2}
\]

\[
P(|\eta - \mathbb{E}\eta| \geq \delta) \leq \frac{\text{var}\eta}{\delta^2} = \frac{C_2^2}{M\delta^2}
\]

So with probability at least \(1 - p\),

\[
|\hat{b}(N, m) - \tilde{b}(N, m)| \leq \frac{C_1}{4\sqrt{M}p} + \frac{C_2}{2\sqrt{M}p}
\]
If at a particular MF \( m \), \( b(N, m) \) is larger than a threshold \( T \), then at this MF no SR algorithm can effectively recover the original HRI:

\[
\text{limit} \leq b^{-1}(T)
\]
Experiments
Estimating the Limits

\( T = 11.1 \) is a large enough threshold.
To take noise into account, $\tilde{g}(N, m)$ should be changed to

$$
\tilde{g}'(N, m) = \int_{h,n} \| h - s(Dh + n) \|^2 p_{h,n} \left( \begin{pmatrix} h \\ n \end{pmatrix} \right) dh dn,
$$

Accordingly,

$$
\tilde{b}'(N, m) = \frac{1}{4} \text{tr} \left[ (I - UD)\Sigma(I - UD)^t \right] \\
+ \frac{1}{4} \text{tr} \left( U\Sigma_n U^t \right) + \frac{1}{4} \| (I - UD)\tilde{h} - U\tilde{n} \|^2.
$$
Future Work and Open Problems

- Tighter upper bound of the limits
- Limits of SR algorithms for specific image classes
- How to represent and incorporate the prior more effectively?
- How to make the algorithms scalable with the MF?
- What is the relationship between the SR performance and the training samples?
  - How to choose optimal training samples?
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References

